Stats - Fundamentals of hypothesis testing Part 1

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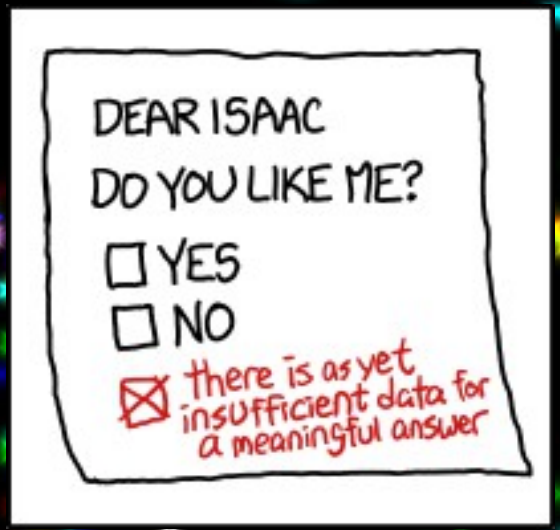
Record Attendance

Any Questions About Any Maths At All?



Do Most Of Us Usually Arrive On Time?

- Last time: how we can estimate the mean arrival time of students to this tutorial?
- New question: do most of us usually arrive on time?
- How do we answer this question? What's the procedure?

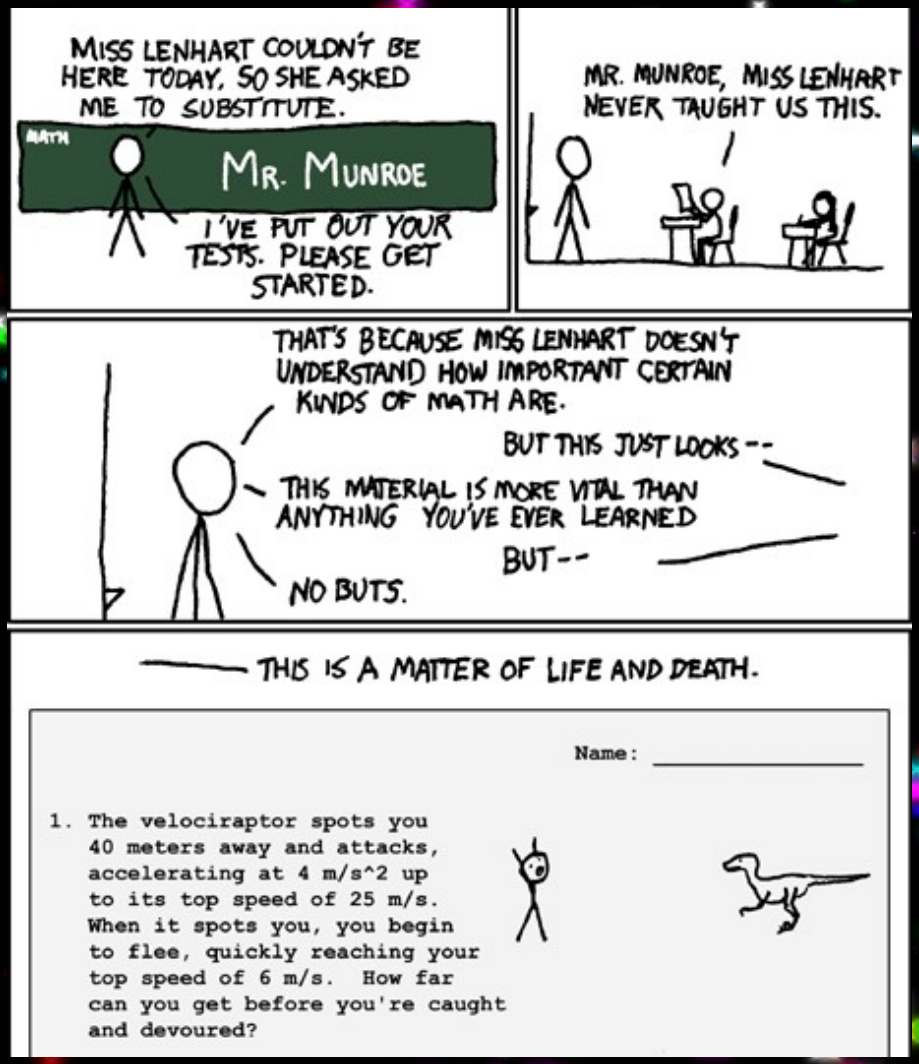
Do Most Of Us Usually Arrive On Time?

- Null hypothesis: most of us usually arrive on time
- In a form we can test: the mean arrival time is before the beginning of the tutorial
- The sample statistic corresponding to the population mean arrival time is the average arrival time to this tutorial
- Null hypothesis says mean arrival time is before start time

Do Most Of Us Usually Arrive On Time?

- Null hypothesis says our sample mean arrival time should be at or before start time
- At 10% level the rejection region is the 10% area in the upper tail
- Does our sample mean fall into this region?
- For the sampling distribution: must estimate standard deviation from sample, sample is small ($n < 30$) so use t distribution
- Using the appropriate row of the t-table, find critical t-value for limit of rejection region, compare with sample mean's t-value

Review the Lecture



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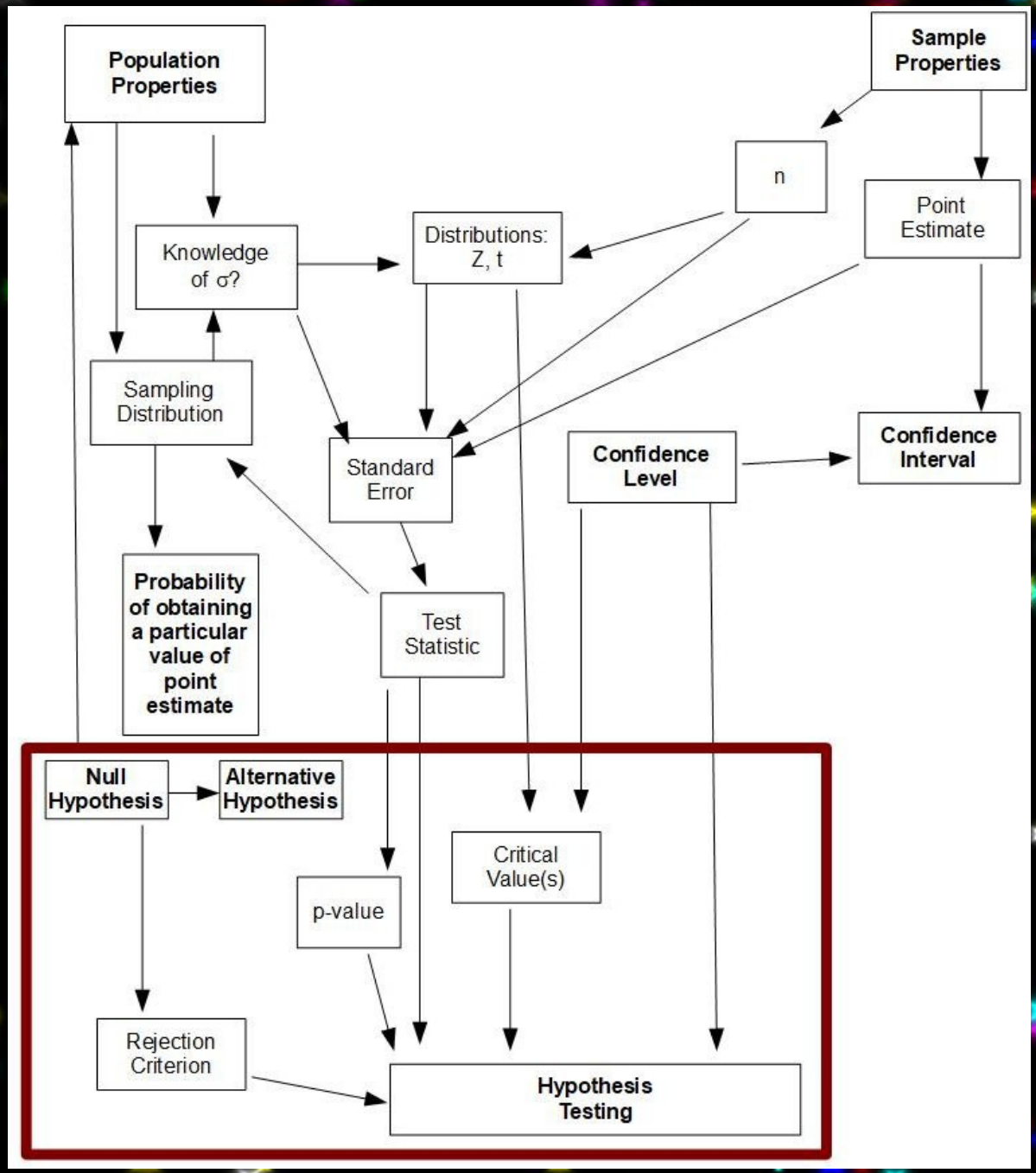
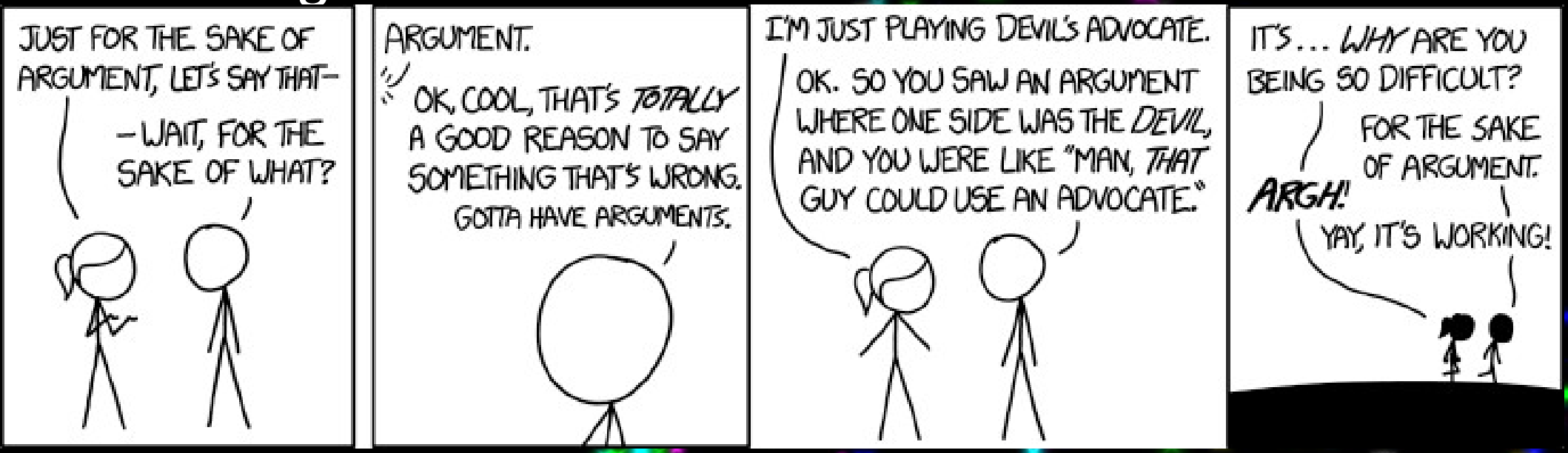
Where Are We Now?

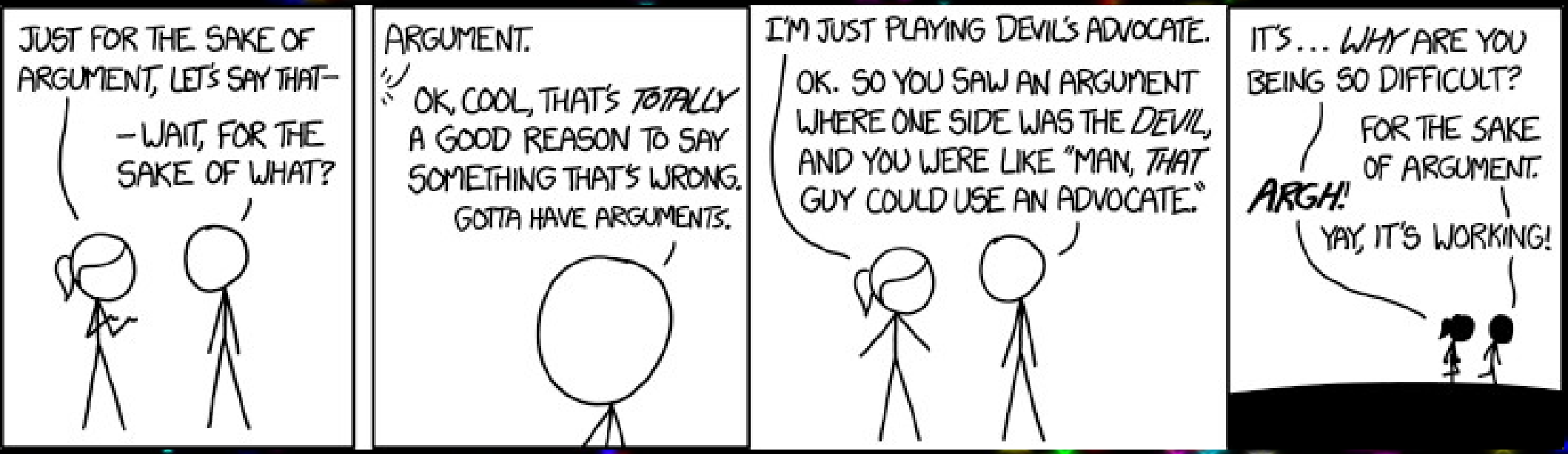


The Sake of Argument

- What does this have to do with hypothesis testing?



The Sake of Argument

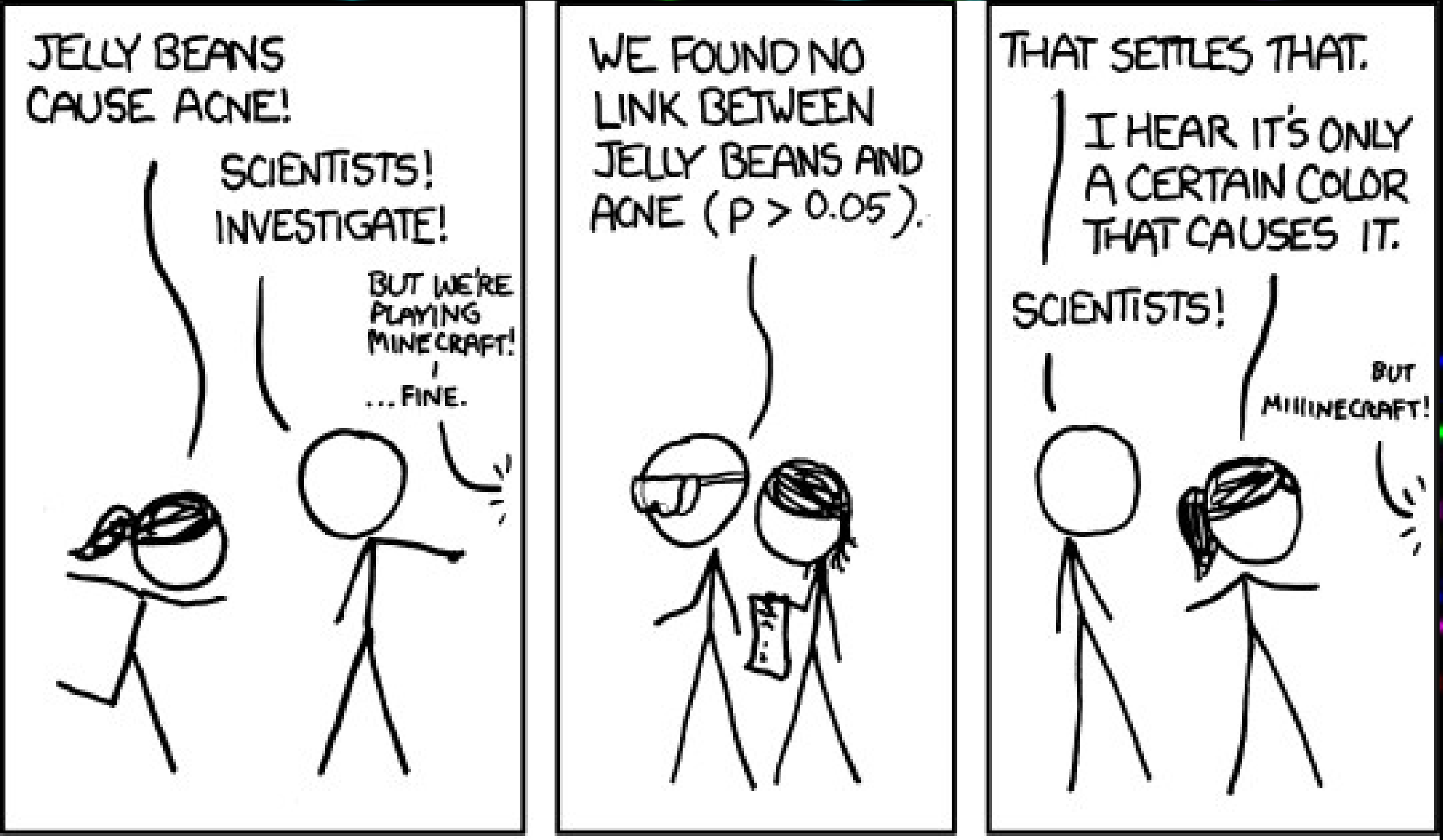


Title text: 'It's not actually... it's a DEVICE for EXPLORING a PLAUSIBLE REALITY that's not the one we're in, to gain a broader understanding about it.' 'oh, like a boat!' '...' 'Just for the sake of argument, we should get a boat! You can invite the Devil, too, if you want.'

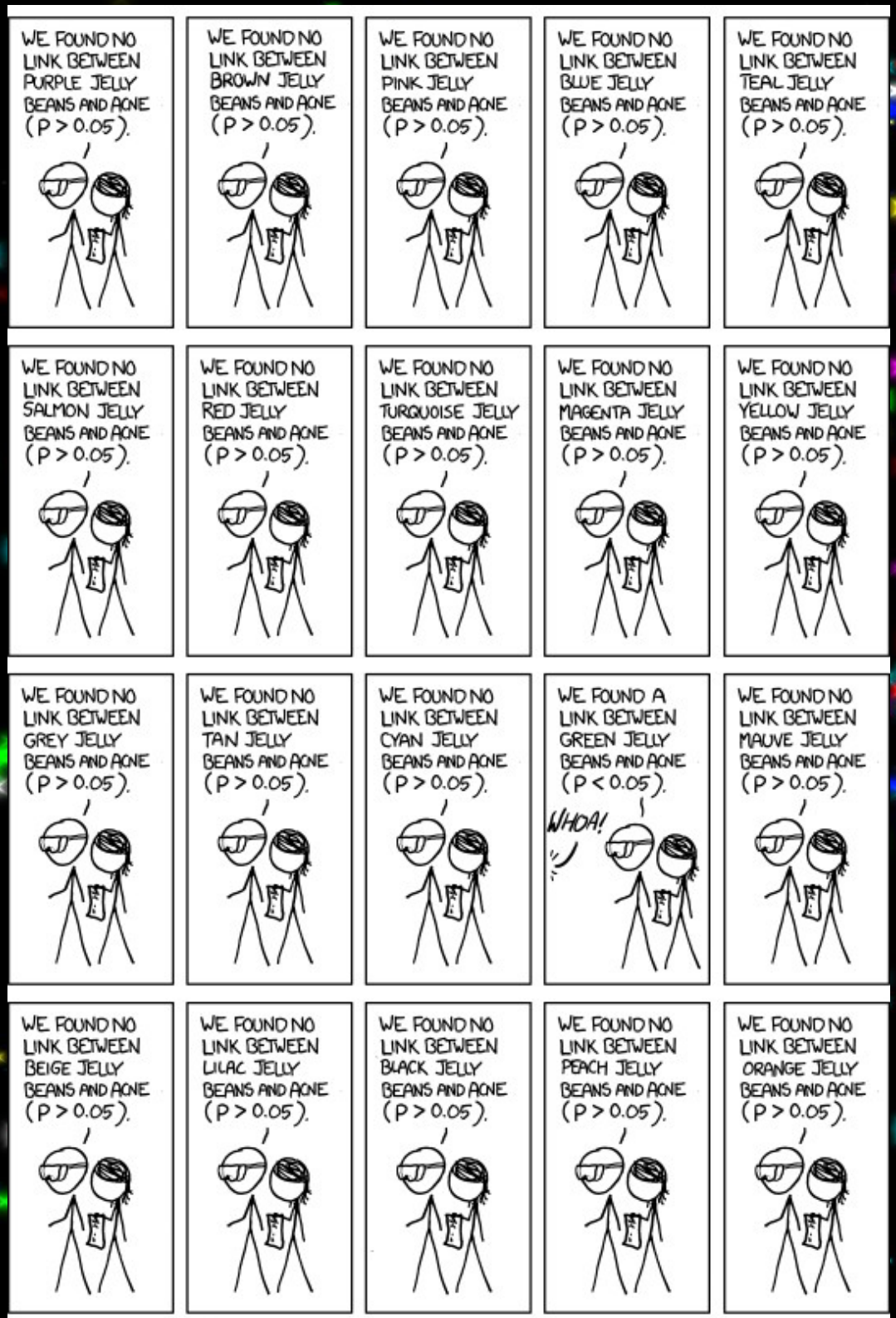
The Meaning of Statistical Significance – '*Show me you're real*'

- Statisticians don't accept results at face value
- A result might not be “real” or “statistically-significant”
- '*Crack, baby crack – show me you're real*'
  - “Cracked Actor” (Aladdin Sane) – David Bowie
- Recall the arrival time example: why do we not necessarily reject the null hypothesis (that we mostly arrive on time) if the sample mean arrival time is after the tutorial start time?

“Significant” – Part 1



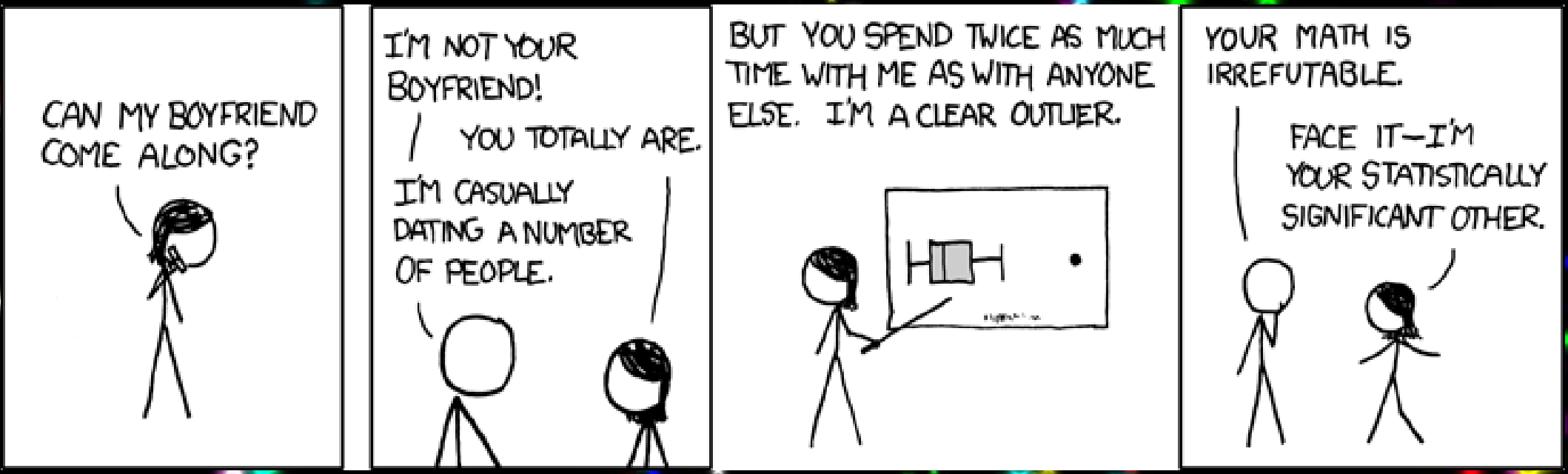
"Significant" – Part 2



The Meaning of Statistical Significance

- Answer: because we might have been “lucky”: the result might have been a fluke
- It might not be statistically-significant
- This is an example of how statisticians think differently to normal people
- So how do we decide whether a result is statistically-significant?

Determining Statistical Significance



The Meaning of Statistical Significance

- Answer: when the result deviates by more than some distance from what we expect
- Determining this distance is what this topic is all about!

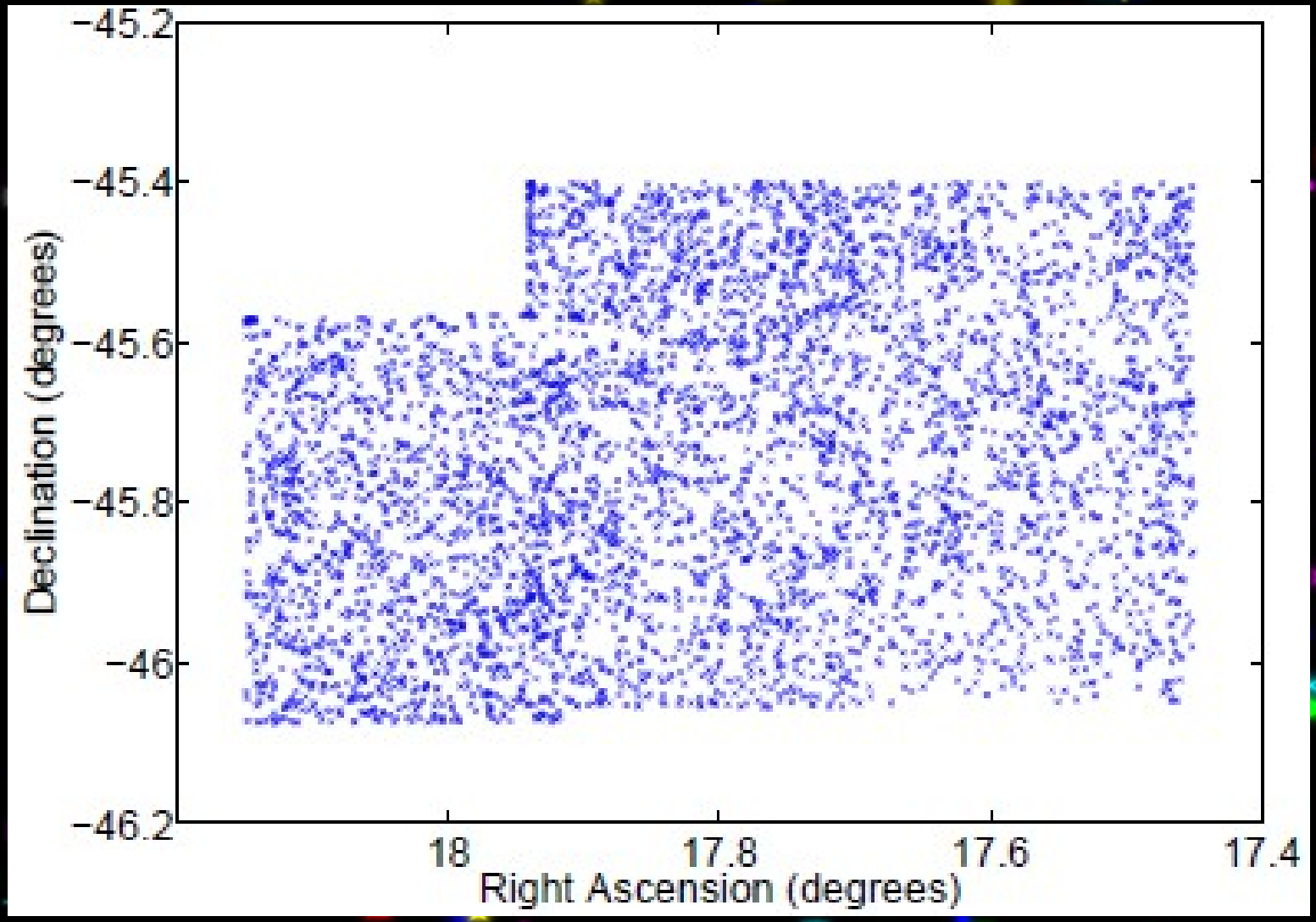
Confidence Level vs Level of Significance

- What's the difference between these two terms?

Confidence Level vs Level of Significance

- Answer: A confidence level is how confident we can be that the true value lies within a certain range
- But a level of significance is the significance a result must at least have to reject the null hypothesis

Problem: Cluster-Finding



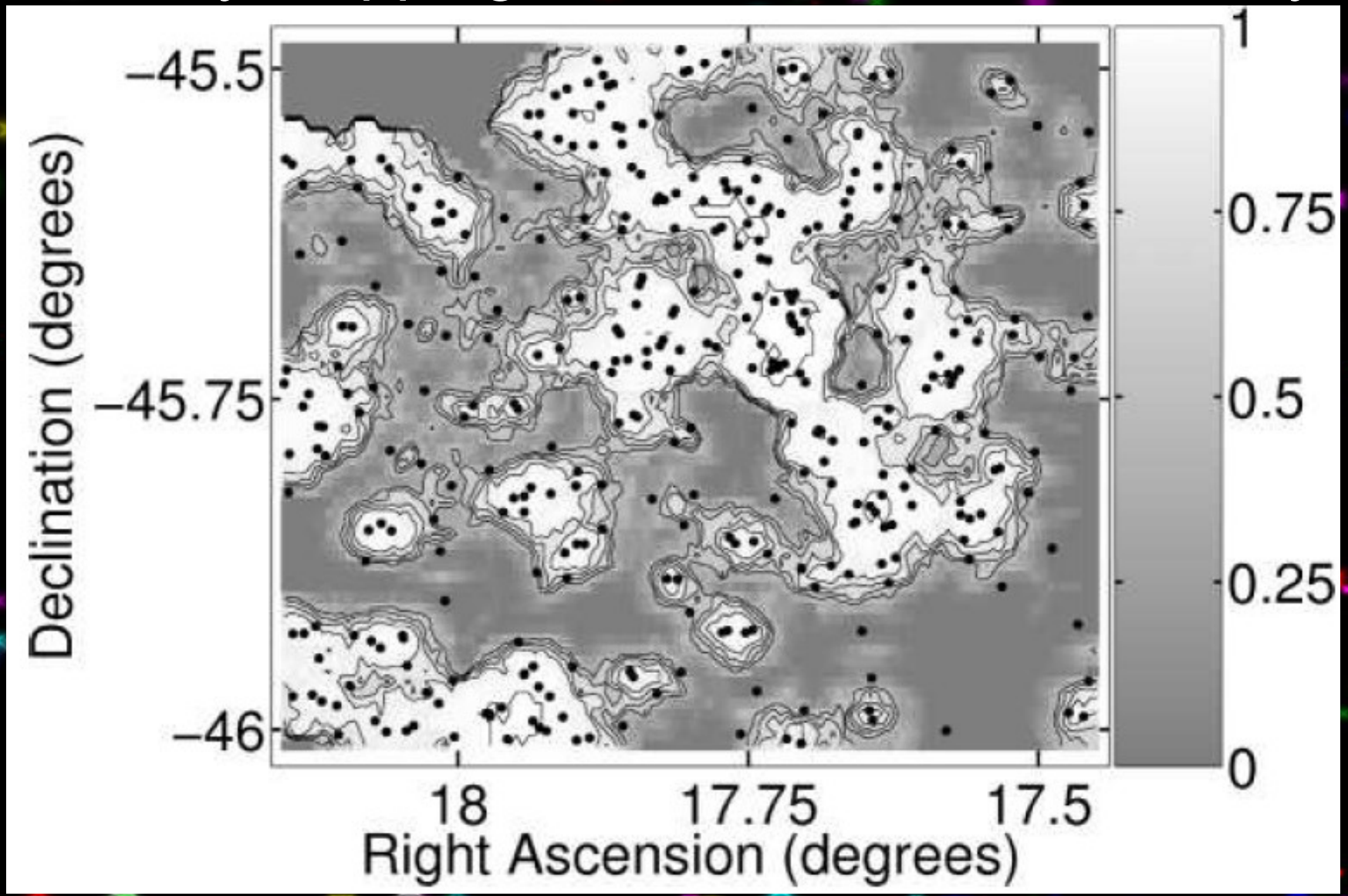
- Want to teach a computer to find the clusters automatically
- Any ideas how?

Testing a Hypothesis at Each Location on the Sky

- Null hypothesis: there is no cluster here
- Alternative hypothesis: there is one
- At each location, we ask: what is the probability of obtaining the observed density of galaxies or more, given the mean density?

Cluster Finding with Statistics

- Probability mapping: a tool to tell us where they are



Quick Example

- Suppose the mean density of galaxies is 0.2, and at a particular location we find a galaxy density of 0.4
- This density at this location is determined using 6 galaxies
- The standard deviation of the galaxy density is 0.2
- Let's assume that the galaxy density from location to location is normally-distributed
- At 10% level, have we found a cluster here?

Quick Solution

- Population standard deviation known so we use Z- distribution
- This is a one-tail test, so rejection region is in the upper tail and has an area of 0.1
- Converting the observed density to Z:
  - $Z = (0.4-0.2) / (0.2 / 6^{0.5})$
  - $Z = 2.4495$

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Quick Solution

- For cumulative probability of 0.9, Z = 1.28 – this is our critical value and the lower limit of the rejection region
- So our Z of 2.45 is in the rejection region and we reject the null hypothesis (that there is no cluster)
- So we have discovered a cluster!

How We Actually Did This

- It's more complicated than that, as usual
  - Link to my thesis

"Who Can I Be Now?"

- Statisticians don't accept results at face value
- A result might not be “real” or “statistically-significant”
- There are often competing hypotheses, each with people who believe them to be real, because people have various points of view
- But with enough data, a statistician has the means – the “sight” to determine which one is actually real
- And so the statistician helps the truth to be real to everyone else

"Who Can I Be Now?"

Everybody feels that everything is real

Anybody's point of view

But even in my life, I knew you found your sight

And nothing would be quite the same

Please help me

Who can I be now? You found me

Who can I be? Fell apart, you found me

Now can I be now? You found me

Now can I be real?

- David Bowie, "Who Can I Be Now?" (*Young Americans*, 1975)

Questions, suggestions fopr next time:
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These slides available from the usual place

Any Questions About Any Maths At All?

Credits

- All comics from xkcd, the webcomic of romance, sarcasm, math and language
  - https://xkcd.com/
- Any songs were by David Bowie